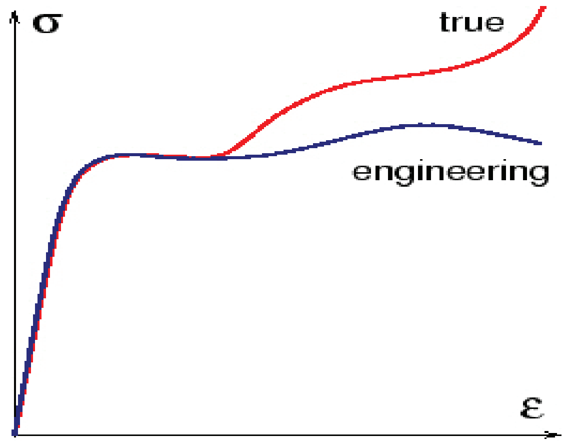
True Stress and True Strain

- For **engineering stress (σ)** and **engineering strain (ε)**, the original (gauge) dimensions of specimen are employed. However, length and cross-sectional area change in plastic region. **True stress (σ')** and **true strain (ε')** are used for accurate definition of plastic behaviour of ductile materials by considering the actual (instantaneous) dimensions.



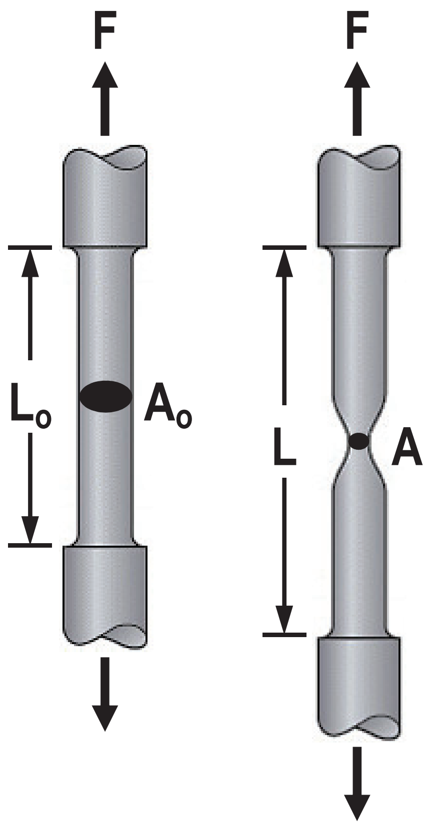
- **True stress (σ')** is the force divided by the actual area: $\sigma' = F/A$
- By the constancy of volume: $V = A * L = A_o * L_o$, we can obtain:

$$\sigma' = \frac{F}{A} = \frac{F*L}{A_o*L_o} \quad \& \quad \varepsilon = \frac{L-L_o}{L_o} = \frac{L}{L_o} - 1 \;\Rightarrow\; \sigma' = \frac{F}{A_o}(1+\varepsilon) = \sigma(1+\varepsilon)$$

- **True strain (ε')** is change in length with respect to the instant length:

$$\varepsilon' = \int_{L_0}^{L} dL/L = \ln(L/L_o) \quad \& \quad L = (1+\varepsilon)L_o \;\Rightarrow\; \varepsilon' = \ln(1+\varepsilon)$$

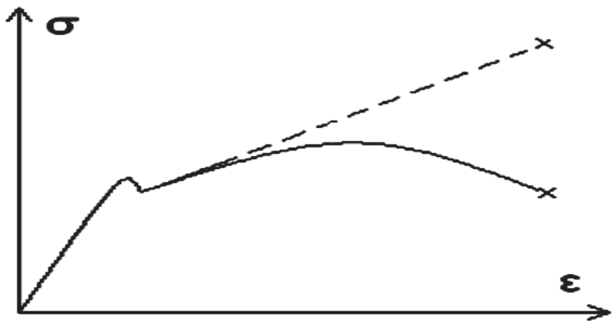
- True strain can also be in terms of reduction in area: $\varepsilon' = \ln(A_o/A)$
- This can be rewritten for cylindrical specimens: $\varepsilon' = 2\ln(D_o/D)$



True Stress and True Strain

- Fig. 21 shows conventional vs true stress-strain diagrams for two different steels in which the portion beyond ultimate strength is significant. When necking starts, only way to define the gauge length is to measure it.

- However, if loads at ultimate and fracture points are recorded, this portion can be approximated by a straight line using the fracture area.



- Thus, true strain at point of fracture is determined by equation below:

$$R_A = 1 - e^{-\varepsilon'_f}$$

R_A : reduction in area

e : natural logarithm

ε'_f : true fracture strain

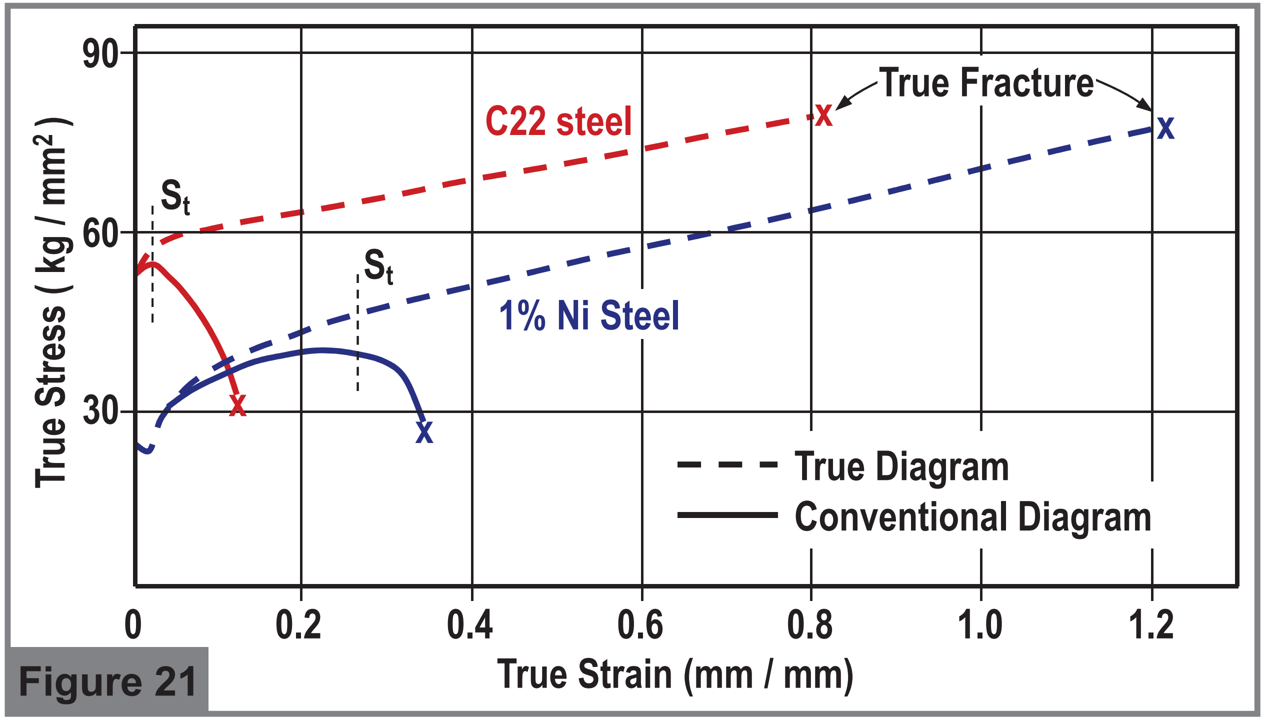


Figure 21

True Stress and True Strain

- There is an approximate linear relationship between true stress and true strain when plotted on log-log scale, as shown in Fig. 22 for various steels.
- For many materials, the correlation between true stress and true strain has been found to be approximately represented by equation below:

$$\sigma' = K * (\varepsilon')^n$$

K : strength coefficient

n : strain hardening exponent

(i.e. slope of log-log plot)

n = 0 (perfectly malleable solid)

n = 1 (elastic solid)

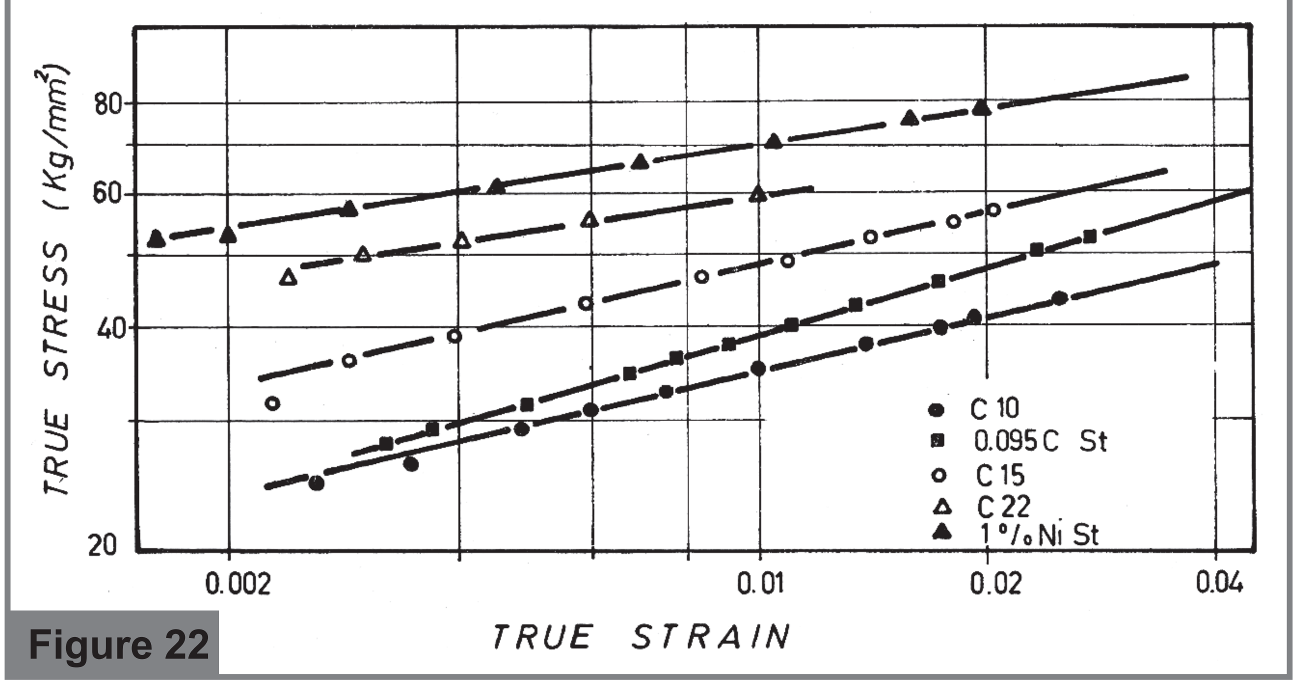


Figure 22

➤ **Alloy content**, **heat treatment** and **fabrication (production) process** are the variables affecting tensile and other properties of steel. The following factors influence the selection procedure of steels:

a. **Rigidity** is purely a design problem. Elastic modulus of steels falls within the range of 19.6 - 20.1 $*$ 10^4 kg/mm^2, regardless of composition or form.

b. **Yield and tensile strength** of carbon steels are strongly affected by their carbon content regardless of alloy content. *Increasing carbon content will increase yield and tensile strength, but decrease ductility* (see Fig. 23).

Some designers use tensile strength while others use yield point. Whichever used is modified by **a factor of safety** to allow uncertainties in stress calculations and possible overloads.

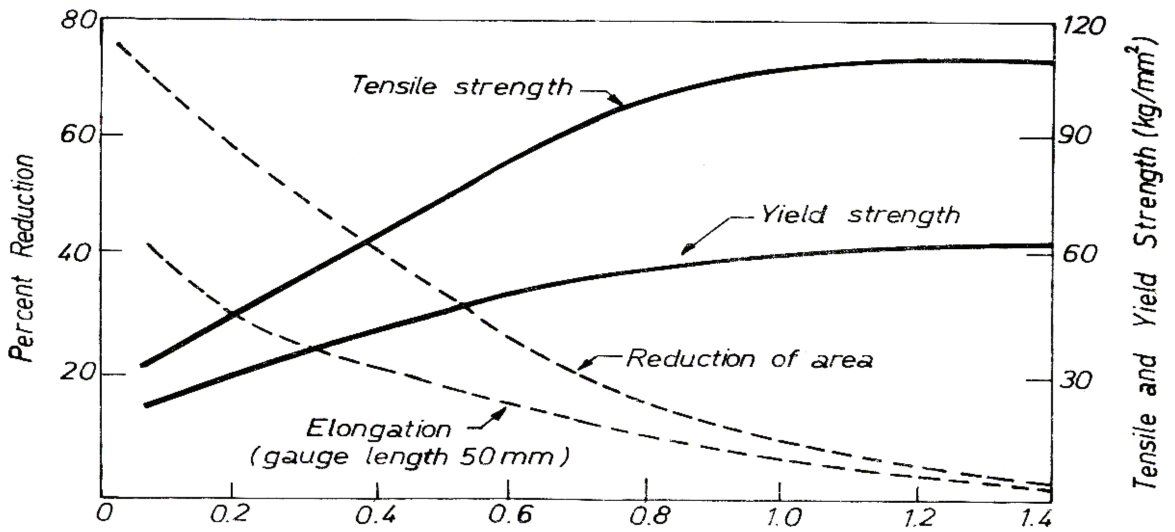


Figure 23

c. **Heat treatment** gives useful and important properties due to **hardenability** in order to obtain *proper combination of strength, ductility and toughness*. Heat-treatable steel for non-cylindrical part must be picked based on **ruling section-size** (i.e. the maximum diameter of round bar in which the specified tensile strength or hardness can be produced by employed heat treatment).

d. **Hardness** is related to strength as: $S_t = f * BHN$ where **f** is a conversion factor (0.33 - 0.36) and **BHN** is the **Brinell Hardness Number**. Steels with the same hardness usually have the same tensile strengths.

e. **Ductility** of carbon steels decreases as hardness increases. *For the same BHN or strength, alloys steels are more ductile than carbon steels.* **An alloy steel** can give better strength, ductility and toughness. Thus, it should be preferred to carbon steel under high stress or impact loading.

f. **Fabrication process** is also significant for the properties of steels. When an annealed steel is **cold worked** (by rolling, wire drawing or elongation in tension), its strength and hardness increase, but its ductility decreases.

Factors Affecting Tension and Compression Tests

- In tension or compression test, the specimen is subjected to a gradually increasing uniaxial load until failure occurs.

- The test is fundamentally dynamic, but due to the low speed of testing involved, it is considered to be quasi-static for all practical purposes.

- There are several factors affecting these tests (e.g. **metallurgical factors** as well as **testing and environmental conditions**).

- Such factors are mainly grouped into two categories:

  1. Variables related to **test specimen** (size and shape effect)
  2. Variables related to **testing machine** (strain rate, rigidity of machine, load and extension measuring device, gripping arrangement)

Variables Related to Test Specimen

- In theory, if a material is of uniform quality, the geometrically similar size of specimens would not affect the test results considerably. However, in practice, mechanical properties change with size.
- This is called **"size effect"**, which is observed in fatigue and brittle fracture according to the statistical distribution of defects in microstructure.
- Different sizes for the same material would give different property values *(as in table)*.

Steel	Size*	Tensile** Strength	Yield** Strength	%Elong.***	Condition
C1020	13	45.50	33.6	36	As rolled
	12.7	45.15	35.18	39.3	Normalized, heated to 927°C, air cooled
	25.4	44.80	35.18	35.8	
	50.8	44.45	32.38	35.5	
	101.6	42.00	28.53	36.0	
	12.7	90.30	50.40	11.4	Carburized at 913°C for 8 hours, reheated to 774°C, quenched in water, tempered at 177°C.
	24.4	62.30	37.80	23.0	
	50.8	52.85	30.63	31.3	
	101.6	49.88	30.28	33.0	
C5150	25.4	68.60	36.23	22.0	Annealed
	12.7	91.70	57.05	21.0	Normalized, heated to 871°C, air cooled.
	25.4	88.38	53.74	20.7	
	50.8	86.10	50.75	20.0	
	101.6	85.40	44.10	18.2	
	12.7	111.13	101.68	16.4	Oil quenched from 829°C, tempered at 538°C.
	25.4	107.10	92.23	17.0	
	50.8	92.40	67.73	18.5	
	101.6	87.50	60.03	20.0	

* *Round specimens, diameters in mm.*
** *kg/mm*^{2}
*** *Measured over a gauge length of 50.8 mm.*

Variables Related to Test Specimen

- **Shape** also affects the results. Fig. 24 shows standard tensile test specimens (**TS 138**).

- In case of compression tests, specimens of circular section are used for uniform straining. Height-to-diameter ratio (**h/d**) is important to avoid buckling and ensure free shear plane in case of brittle materials:

Type	h/d	Purpose
Short	0.9	Testing bearing materials
Medium	3	General purpose uses
Long	8-10	To determine stiffness

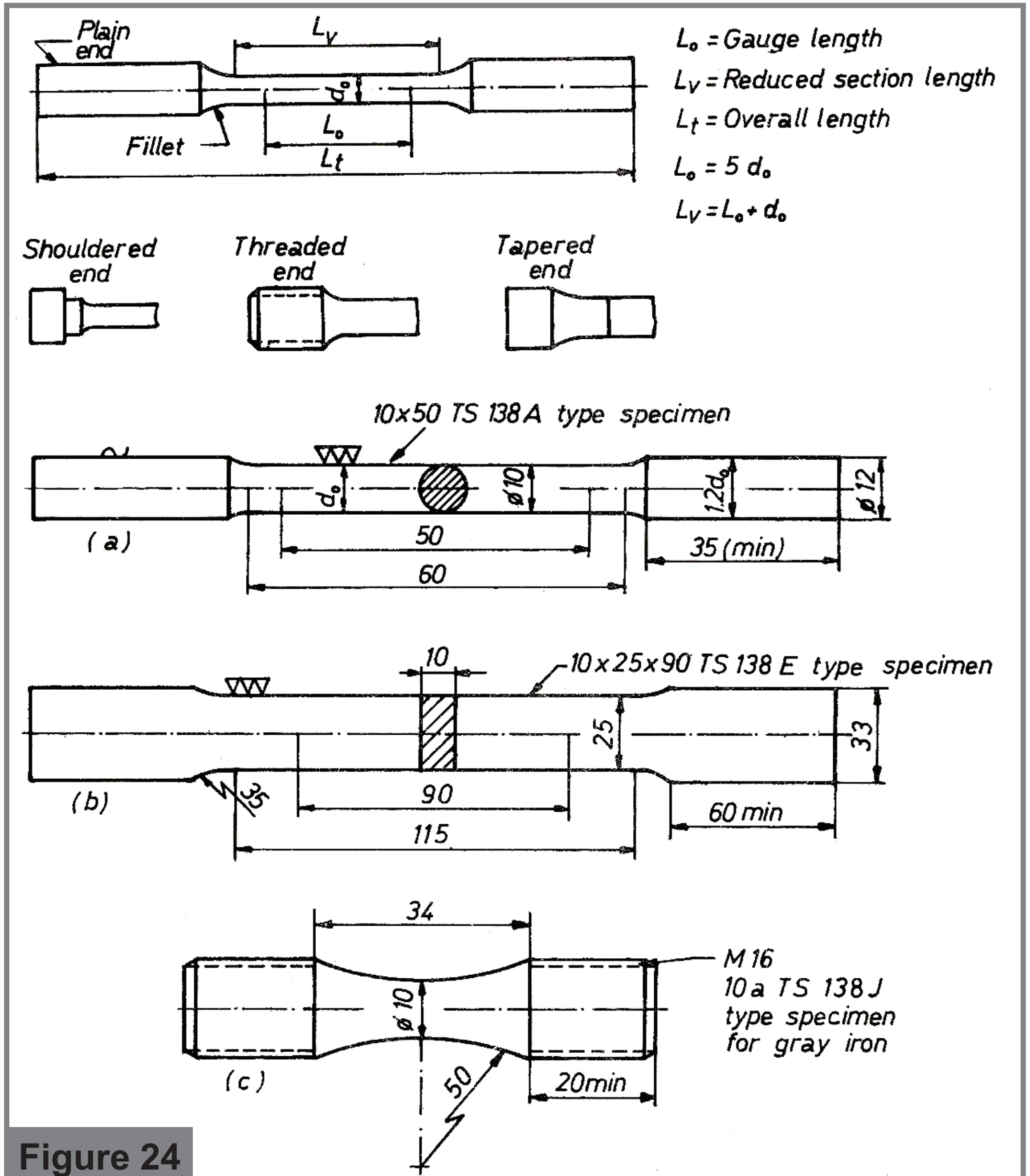


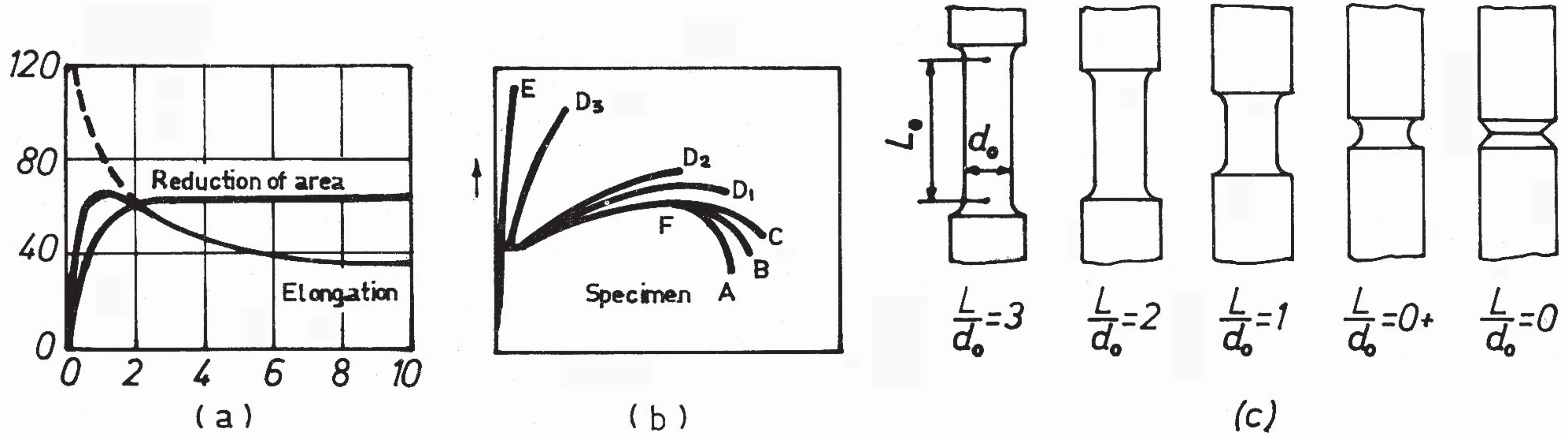
Figure 24

Variables Related to Test Specimen

- Fig. 25a shows how ductility and shape of stress-strain diagram varies with **L_o/d_o ratio**. The reduction of area is independent of L_o/d_o for values of > 2 and drastically reduced for values of < 2. This is called **"notch sensitivity"**. *The term "notch" implies any kind of stress concentration effect.*

- Small ratios convert ductile type of stress-strain curves to brittle type (as in Fig. 25b). Stress concentrations are unimportant where static loads are acting on ductile materials. However, brittle materials have limited capacity of plastic flow, so a notch adversely affects strength causing sudden failure.

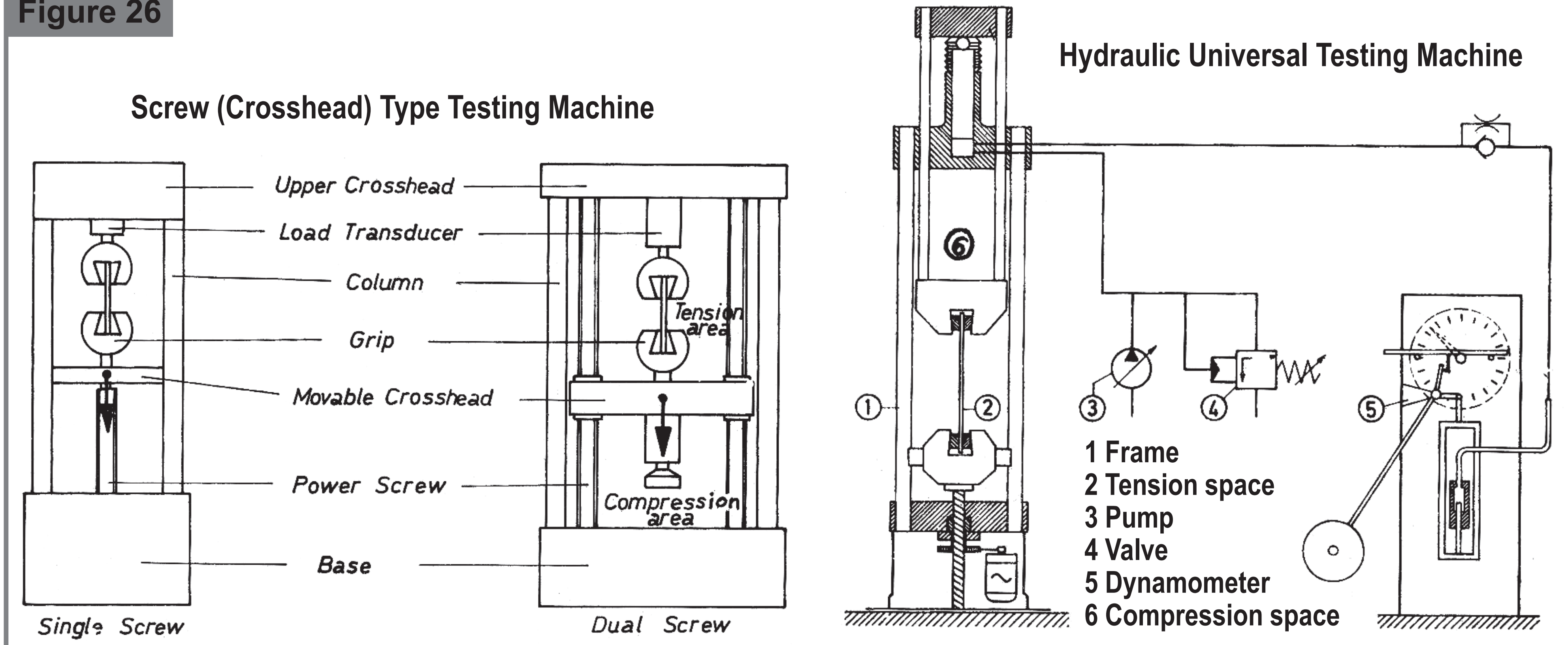
Figure 25



Variables Related to Testing Machine

- Fig. 26 illustrates two commonly used testing machines by which tension and compression tests in uniaxial direction can be performed. Electronic version of these testers are also available.

Figure 26



- The factors related to the testing machine are: strain rate and strain history, rigidity of machine, load and extension measuring device, gripping devices.

- **Strain rate (dε/dt)** is related to the speed of gripping heads. Fig. 27 shows the effect of strain rate *(curve shifts upwards at higher speeds)*.

- **ASTM**, **Turkish** and **DIN** standards for appropriate strain rates to be used in testing various materials are given in tables below:

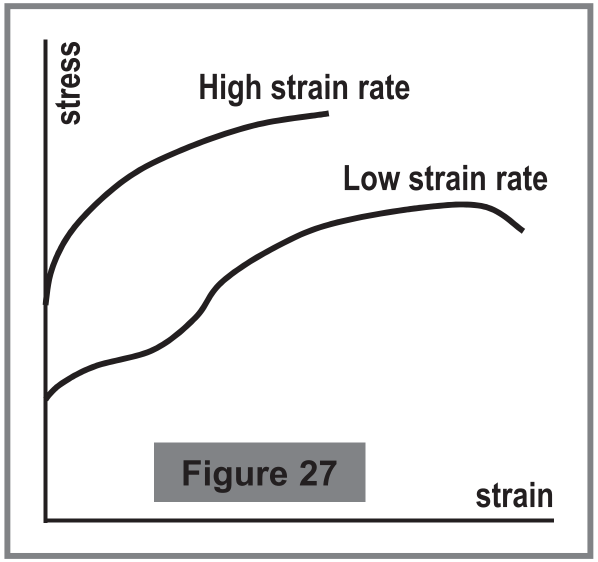


Figure 27

Various ASTM requirements on strain rate

Material Tested	ASTM Ref.	Max. Crosshead Speed (mm/min) To yield	To ultimate	Load rate
Metallic materials*	E8	0.062 mm per mm of gauge length	0.5 mm per mm of gauge length above 10 kg/mm^2	Maximum 70 $kg/mm^2/min$ to yield
Steel prod.*	A370			
Gray CI	A48			
		Specified Grip Speed (mm/min)		
Plastics	D638	1.30	5 - 6.35	1.32 - 1.4 kg/s
Hard rubber	D530			
Soft rubber	D142		500	

** The values are also recommended in TS 138*

Strain rates for metallic materials (DIN)

For determination of upper yield point	dσ/dt dε/dt	1 (kg/mm^2s) 0.3 (% / minute)
For determination of lower yield point	dε/dt	10 (% / minute)
For determination of tensile strength (before reaching max. force)	dε/dt	40 (% / minute)

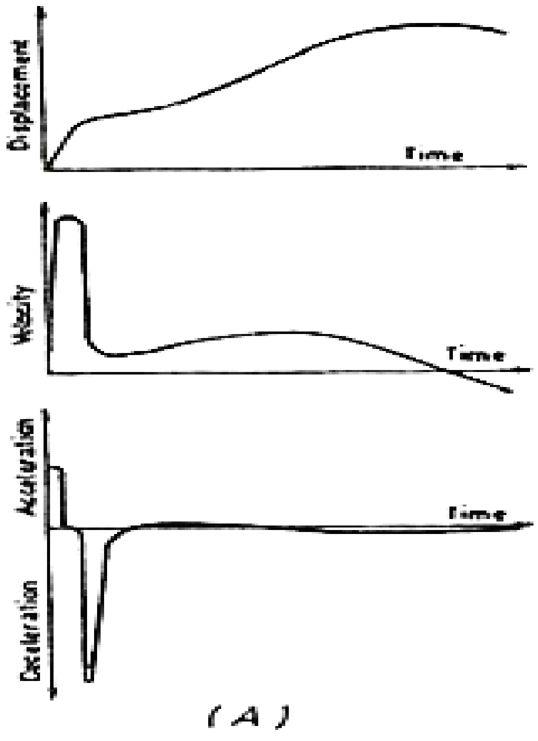
- **Strain history** also modifies the stress-strain diagram. When a material is subjected to **a cycle of loading and unloading**, some energy is dissipated by the specimen, which is called **"hysteresis effect"**. It is an important consideration in the treatment of "anelasticity" and "fatigue".

- **Rigidity of machine** contributes to the deformation measurements. Under loading, not only the specimens but also components of machine elongate. Due to **the deformation of machine itself**, it is also difficult to maintain constant strain rate of the specimen, which again affects the test result.

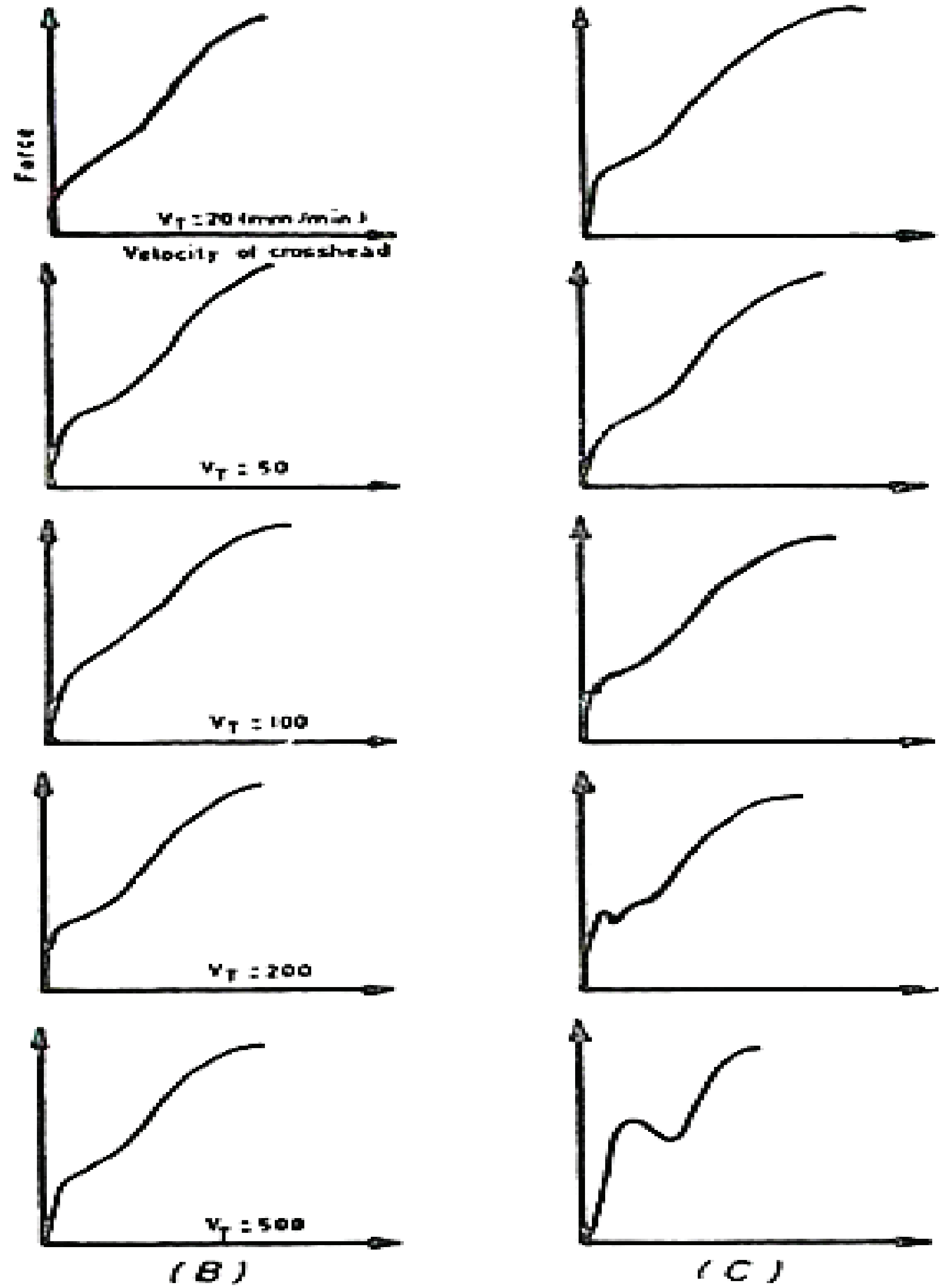
- **Load and extension measuring device: Mechanical systems with high inertia** will cause changes in measurements especially at high speeds when acceleration comes into play. In contrast, **electronic systems with negligible inertia** always provide the same measurement at all speeds. *Hence, it is better to use low inertia measuring devices for accurate results.*

- Similar to rigidity of machine, **gripping devices** also must be **stiff and rigid** *(i.e. no bending effect on the specimen should occur)*.

Variables Related to Testing Machine



Material : Polyester.
$L_o = 50mm$, $d_o = 0.8\ mm$



Inertia effect on a pendulum type measuring system (A). Comparison of force-displacement curves obtained by electronic (B) and mechanical (C) force measuring systems (ZWICK).